Formal Specification of Abstract Datatypes
Exercises 4+5 (June 16+July 7)

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The result for each exercise is to be submitted by the deadline stated above via the Moodle interface as a .zip or .tgz file which contains

- a PDF file with
  - a cover page with the title of the course, your name, Matrikelnummer, and email-address,
  - the content required by the exercise (specification, source, proof),
- (if required) the CafeOBJ (.mod) file(s) of the specifications.

Exercise 4: Specification of Queues

A "queue" is a "First In/First Out" data structure with operations `empty` (the queue without any elements), `isempty` (is the queue empty?), `enqueue` (add an element to the tail of the queue), `dequeue` (delete an element from the head of the queue), `head` (return the element at the head of the queue). Queue elements are assumed to be values of the unspecified sort `Elem`.

1. Write a loose specification with (free) constructors of the abstract datatype `Queue` in a logic of your choice.
2. Write an initial specification of the abstract datatype `Queue` in conditional equational logic.
3. Compare the specifications and discuss their differences. Are the specifications strictly adequate with respect to the classical algebra of queues?
4. Implement the initial specification in CafeOBJ (using `Nat` for `Elem`) and test it with a couple of sample reductions.

Exercise 5: Strict Adequacy of Specification

Take above initial specification of `Queue` (where `Elem` is replaced by a sort `Nat` with operations `0` $:\rightarrow$ `Nat`, `s` : `Nat` $\rightarrow$ `Nat` which are assumed to strictly adequately specify the algebra of natural numbers). Show that this specification is strictly adequate with respect to the classical algebra of queues using the proof technique of characteristic term algebras.

The domain *Queue* of the classical algebra of queues can be defined as

$$
\begin{aligned}
&\mathit{Queue} := \bigcup_{i \in \mathbb{N}} Q_i \\
&Q_i := \mathbb{N}_i \rightarrow \mathbb{N}
\end{aligned}
$$

where $\mathbb{N}_i$ is the set of the first i natural numbers. Consequently, Q_i is the set of finite sequences (of natural numbers) of length i. By this definition, to show $\forall q \in \mathit{Queue} : P(q)$, it suffices to show $\forall i \in \mathbb{N}, q \in Q_i : P(q)$ by induction on i.

1. Based on above hint, give a full definition of the algebra of queues with functions for the operations in `Queue`.
2. Define a characteristic term algebra for `Queue` and prove that it is indeed characterstic.
3. With the help of the characteristic term algebra, prove the strict adequacy of `Queue` with respect to the algebra of queues.